Finite Automata: in principle

Suppose that Σ is a finite alphabet.

1. Briefly describe how a ***deterministic finite automaton*** (DFA) over Σ acts, in terms of an edge-labeled digraph:
   (a) What in the graph corresponds to the finite set of ***states*** of the DFA?
   (b) What in the graph corresponds to the ***transitions*** of the DFA, and what rules apply to those transitions?
   (c) How do we mark the unique ***start state*** of the graph?
   (d) What special designation can each state have (or not have)?
   (e) Given a *string* of characters from Σ, describe how the DFA operates one character at a time, and what it means for the DFA to ***accept*** or ***reject*** the string.

2. Taking a step back, as an input/output machine, what does a DFA over Σ take as input? What are its possible outputs? Viewing this as a *function*, what are its domain and codomain?

3. Briefly explain why each string $x \in \Sigma^*$ must be either accepted or rejected (but not both!) by a DFA.
   What do we mean by the ***language accepted by*** a DFA?

4. Taking a further step back, how can we think of this whole process as giving us a function from the set of all DFA's to $\mathscr{P}(\Sigma^*)$?

. . . and in practice

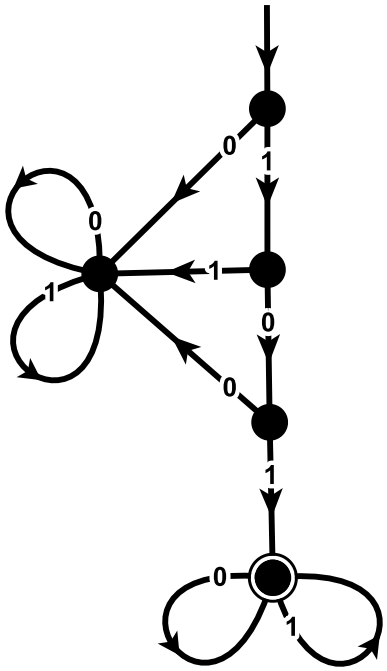
5. Consider the DFA labeled A on the back of this page.
   (a) Will this DFA accept or reject: (i) a string x that starts with `101`; (ii) a string y that starts with `100`; (iii) a string z that starts with `11`; (iv) a string w that starts with `0`?
   (b) Find a regular expression that produces the language accepted by A.

6. Consider the DFA labeled B on the back of this page.
   (a) Will this DFA accept or reject: (i) the string ε; (ii) the string `1000100`; (iii) the string `00100100`; (iv) the string `11100011`?
   (b) Find a regular expression that produces the language accepted by B.

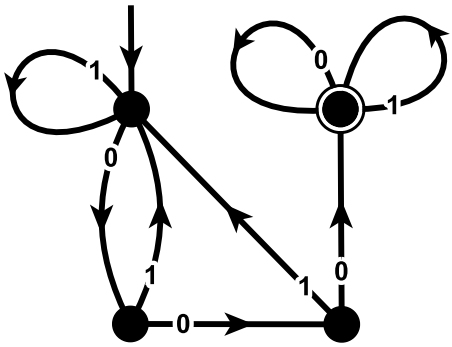
7. Consider the DFA labeled C on the back of this page.
   (a) Will this DFA accept or reject: (i) the string ε; (ii) the string `001`; (iii) the string `001001`; (iv) the string `00100100`?
   (b) Find a regular expression that produces the language accepted by C.

8. Consider the DFA labeled D on the back of this page.
   (a) Will this DFA accept or reject: (i) the string ε; (ii) the string `1111`; (iii) the string `00011`; (iv) the string `001110`?
   (b) Find a regular expression that produces the language accepted by D.

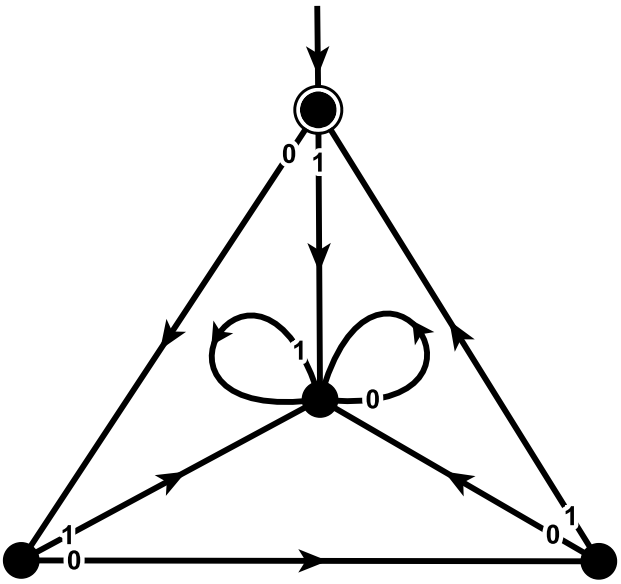
DFA "A"



DFA "B"



DFA "C"



DFA "D"